AEPA®

ARIZONA EDUCATOR PROFICIENCY ASSESSMENTS®

STUDY GUIDE

10 Mathematics

**This AEPA test was replaced by a NES test.
Examinees may continue to find this study guide useful as they prepare for the NES, as the previous AEPA test may have covered objectives and content similar to the NES test.**

AZ-SG-FLD010-03

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GENERAL INFORMATION ABOUT THE AEPA® AND TEST PREPARATION

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General Information About the AEPA and Test Preparation

PART 2:
FIELD-SPECIFIC INFORMATION
Field 10: Mathematics

INTRODUCTION

This section includes a list of the test objectives, practice questions, an answer key for the selected-response questions, and a list of preparation resources.

Test objectives. As noted earlier, the test objectives are broad, conceptual statements that reflect the knowledge, skills, and understanding an entry-level educator needs to practice effectively in Arizona schools. The list of test objectives for each test field is the *only* source of information about what a specific test will cover and therefore should be studied carefully.

Practice questions. The practice questions for the selected-response and performance assignment sections included in this section are designed to give you an introduction to the nature of the questions included in the AEPA tests. The practice questions represent the various types of questions you may expect to see on an actual test; however, they are *not* designed to provide diagnostic information to help you identify specific areas of individual strength or weakness or to predict your performance on the test as a whole.

When you answer the practice questions, you may wish to use the sample answer sheet and sample Written Response Booklet provided in Part 1 to acquaint yourself with these materials. Use the answer key located after the practice questions to check your answers. Sample responses are provided immediately following each written performance assignment. The sample responses in this guide are for illustrative purposes only. Your written response should be your original work, written in your own words, and not copied or paraphrased from some other work.

To help you identify how the test objectives are measured, the objective statement to which the question corresponds is listed in the answer key. When you are finished with the practice questions, you may wish to go back and review the entire list of test objectives and descriptive statements for your test field.

Preparation resources. The list of preparation resources has been compiled to assist you in finding relevant materials as you prepare to take the Mathematics test. This list is to be considered not as complete, but as representative of the kinds of resources currently available. There may be other materials that may be helpful to you in preparing to take the test.

You may also wish to consult a representative from an Arizona educator preparation program in your area regarding other potential resources specific to this field. Keep in mind that the use of these materials does not guarantee successful performance on the test.

Field 10: Mathematics

SUBAREAS:

1. Number Sense
2. Data Analysis and Probability
3. Patterns, Algebra, and Functions
4. Geometry and Measurement
5. Trigonometry and the Conceptual Foundations of Calculus
6. Mathematical Structure and Logic

NUMBER SENSE

0001 Understand principles and concepts related to integers, fractions, decimals, percents, ratios, and proportions and their application to problem solving.

For example: using a variety of models and methods (e.g., manipulatives, estimation, technology) to explore concepts (e.g., absolute value) and patterns and solve problems involving integers, fractions, decimals, percents, ratios, and proportions.

0002 Understand the properties of the real and complex number systems, and solve problems related to their structure.

For example: analyzing and applying properties (e.g., distributive, associative) of the real and complex numbers and their subsystems (e.g., rational numbers, irrational numbers); applying procedures related to order of operations; finding sums, products, powers, and roots of complex numbers; representing complex numbers as vectors; and recognizing the trigonometric form of a complex number.

0003 Understand the principles of number theory.

For example: analyzing properties of divisibility and prime numbers; applying the Euclidean algorithm; and analyzing properties of congruence classes and modular arithmetic.

0004 Understand the principles and properties of discrete mathematics and the application of discrete mathematics to problem solving.

For example: analyzing and applying the properties of finite sets; solving enumeration and finite probability problems; patterns; algorithms and algorithmic thinking; combinatorics; recursive relationships; sequences and series; and modeling and solving problems using the techniques of graph theory.

DATA ANALYSIS AND PROBABILITY

0005 Understand principles and concepts of descriptive statistics and their application to the problem-solving process.

For example: constructing and interpreting tables, charts, and graphs (e.g., line plots, stem-and-leaf plots, box plots, scatter plots) related to descriptive statistics; determining and interpreting measures of central tendency and dispersion; interpreting, calculating, and solving problems related to correlations, frequency distributions, and percentile scores; and recognizing the effects of data transformations on measures of central tendency and variability.

0006 Understand the fundamental principles of probability and probability distributions.

For example: applying fundamental axioms of probability; computing theoretical probability; using simulations to explore real-world situations; applying knowledge of connections between geometry and probability (e.g., probability as a ratio of two areas); and using probability models to describe real-world phenomena.

0007 Use techniques related to probability and probability distributions to analyze real-world situations.

For example: modeling and solving problems involving dependent, independent, and mutually exclusive events and conditional probabilities; applying the concept of random variables; and analyzing and applying properties of the binomial, uniform, and normal distributions.

0008 Understand methods used in collecting, reporting, and analyzing data.

For example: evaluating real-world situations to determine appropriate sampling techniques and methods for reporting data; applying procedures for designing a statistical experiment; making appropriate inferences, interpolations, and extrapolations; and solving problems involving linear regression models.

PATTERNS, ALGEBRA, AND FUNCTIONS

0009 Understand algebraic functions, relations, and expressions.

For example: manipulating algebraic expressions; describing and analyzing characteristics of algebraic patterns, functions (e.g., domain, range, continuity), and relations; using qualitative and quantitative graphing; analyzing relationships among tabular, graphic, and algebraic representations of functions; interpreting the graphic representation of a function and its inverse; analyzing how changing parameters affects the graph of a function; and operating with composite functions.

0010 Understand the principles and properties of linear and matrix algebra.

For example: analyzing properties of matrices and determinants; using matrices to solve systems of equations; representing vectors geometrically and algebraically; and finding the matrix of a linear transformation.

0011 Understand the properties of linear and quadratic equations, inequalities, and functions and their application to the problem-solving process.

For example: using linear and quadratic functions to model and solve problems; evaluating the accuracy and applicability of a linear or quadratic model for real-world problems; and solving linear and quadratic systems using geometric and algebraic methods.

0012 Understand radical, exponential, and logarithmic functions and their application to the problem-solving process.

For example: applying radical, exponential, and logarithmic functions to model real-world problems; understanding the use of graphing calculators or computers to find numerical solutions to such problems; evaluating the accuracy and applicability of such models; and recognizing and generating algebraic and geometric representations of functions.

0013 Understand and interpret polynomial, rational, and absolute value functions and relations.

For example: analyzing properties and graphs of polynomial, rational, and absolute value functions; modeling problems involving these functions; and understanding the use of graphing calculators or computers to find numerical solutions to such problems.

GEOMETRY AND MEASUREMENT

0014 Understand principles, concepts, and procedures related to measurement.

For example: applying knowledge of attributes of length, area, volume, mass, capacity, time, temperature, speed, angles, and rotational speed, and the units (metric and customary) and tools used to measure them; applying knowledge of conversions within systems; and using dimensional analysis (unit analysis) to solve problems.

0015 Understand properties of geometric figures and their application to the problem-solving process.

For example: using synthetic geometric concepts (e.g., similar and congruent figures, parallel lines) to solve problems in context; classifying geometric figures (e.g., parallelograms); deducing properties of polygons and circles; and justifying geometric constructions.

0016 Understand and interpret drawings of three-dimensional objects.

For example: identifying and analyzing three-dimensional figures; representing figures in three-dimensional coordinate systems; analyzing perspective drawings, cross sections (including conic sections), and nets (e.g., folding connected squares into a cube); and generating three-dimensional figures from two-dimensional shapes.

0017 Understand the principles and properties of coordinate geometry and the connections between geometry and algebra.

For example: applying geometric concepts (e.g., distance, midpoint, slope, parallel and perpendicular lines) to model and solve problems; analyzing coordinate representations (e.g., rectangular, polar) of geometric figures; analyzing properties of conic sections; and translating between synthetic and coordinate representations.

0018 Understand the principles of vectors and transformational geometry and the application of transformational geometry to the problem-solving process.

For example: applying transformational techniques (e.g., translations, rotations, reflections, dilations, tessellations) to determine graphic and numeric solutions to problems; interpreting figures in terms of transformations (e.g., determining center of rotation, line of reflection, scale of dilation); deducing geometric properties using transformations; and analyzing the algebraic structure of the composition of transformations.

TRIGONOMETRY AND THE CONCEPTUAL FOUNDATIONS OF CALCULUS

0019 Understand techniques used to model and solve problems related to triangles.

For example: applying trigonometric ratios and relationships (e.g., law of sines, law of cosines) to model and solve problems; and deducing properties of polygons and circles (e.g., perimeter and area) using trigonometric ratios and relationships.

0020 Understand the properties of trigonometric functions and identities.

For example: solving problems involving radian measure and trigonometric equations; analyzing the relationship between circular and trigonometric functions; analyzing the graphs of a trigonometric function in terms of amplitude, period, and phase shift; using trigonometric functions to model real-world periodic phenomena; and recognizing relationships among trigonometric functions.

0021 Understand characteristics and applications of the concepts of limit, continuity, and rate of change.

For example: demonstrating an understanding of characteristics and concepts related to limits of algebraic functions; limits of infinite sequences and series; the continuity/discontinuity of functions; applying and interpreting the equation of a secant line; and solving problems involving average rates of change (e.g., average velocity, average acceleration).

0022 Understand the derivative of a function and its applications to the problem-solving process.

For example: recognizing and interpreting the slope of a line tangent to a function as the limit of the secant line; determining maxima, minima, points of inflection, and concavity and applying this information to describe a function; applying the slope of the tangent line to analyze functions; and modeling and solving problems involving the derivative of a function.

0023 Understand the integral of a function and its applications to the problem-solving process.

For example: applying algebraic and geometric techniques to approximate the area under a curve; recognizing the definite integral as the area under a curve; analyzing the relationship between differentiation and integration; evaluating algorithms for integrating a function; and modeling and solving problems involving the integral of a function.

MATHEMATICAL STRUCTURE AND LOGIC

0024 Understand principles of problem solving, and apply varied and efficient problem-solving techniques and strategies and technological tools to explore and solve problems in context.

For example: applying appropriate techniques and multiple strategies (e.g., estimating, guess and check, using a diagram, making a list) to solve problems, including multistep, nonroutine, and open-ended problems; recognizing strengths and limitations of these techniques and strategies; and understanding the process of mathematical modeling and the use of technological tools (e.g., manipulatives, graphing calculators, computers) for modeling and solving problems.

0025 Understand mathematical communication and the use of mathematical terminology, symbols, and representations to communicate information.

For example: communicating mathematical ideas, concepts, and reasoning processes to a variety of audiences; interpreting mathematical symbols and representations; using graphic, numeric, symbolic, and verbal representations to communicate and model mathematical concepts and relationships; and making connections among graphic, numeric, symbolic, and verbal representations.

0026 Understand mathematical reasoning, and apply techniques of mathematical reasoning in varied contexts.

For example: using inductive and deductive reasoning to solve problems; understanding the properties and use of the converse, contrapositive, and inverse of a statement; understanding modes of inquiry and research in mathematics; applying knowledge of the components of a formal argument; using counterexamples to formulate or evaluate arguments; applying techniques for constructing and evaluating an indirect proof; and formulating, evaluating, and justifying conjectures and arguments.

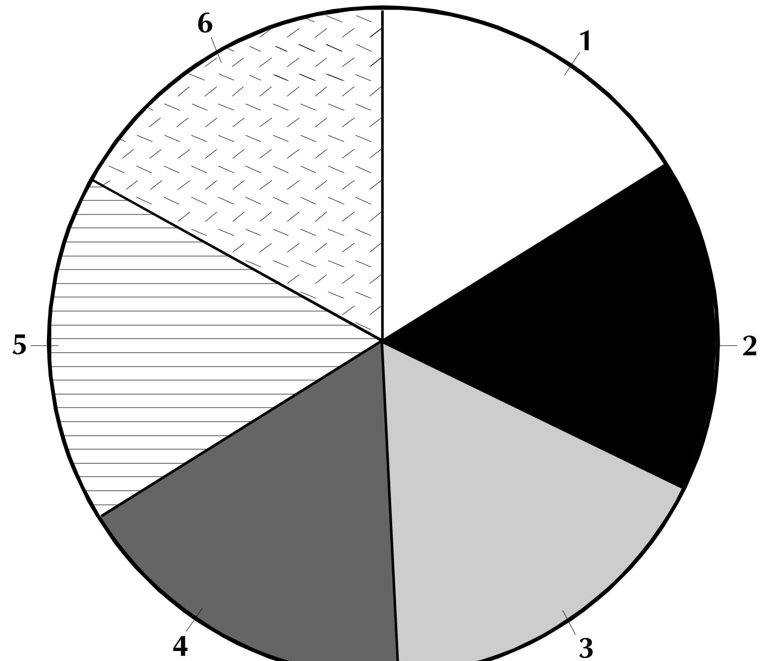
0027 Understand the connections among the domains of mathematics and the relationships between mathematics and other disciplines and real-world situations.

For example: analyzing conceptual connections among the various mathematical domains; recognizing how methods from one or more domains can be used to solve problems in another domain (e.g., using geometric methods to help solve algebraic problems); and recognizing how mathematical concepts and strategies can be used to solve problems in other disciplines and in real-world situations (e.g., analyzing census data, determining the capacity of a container).

0028 Understand the nature and structure of axiomatic systems.

For example: analyzing the relationships among undefined terms, axioms, and theorems; and analyzing the foundations, assumptions, and properties of various geometries (e.g., Euclidean, non-Euclidean) and algebras, and the connections between their mathematical structure.

DISTRIBUTION OF SELECTED-RESPONSE ITEMS ON THE TEST FORM



	Subarea	Approximate Percentage of Selected-Response Items on Test Form
	1. Number Sense	14%
	2. Data Analysis and Probability	14%
	3. Patterns, Algebra, and Functions	18%
	4. Geometry and Measurement	18%
	5. Trigonometry and the Conceptual Foundations of Calculus	18%
	6. Mathematical Structure and Logic	18%

FORMULAS

Formula	Description
$V = \frac{1}{3}Bh$	Volume of a right cone and a pyramid
$A = 4\pi r^2$	Surface area of a sphere
$V = \frac{4}{3}\pi r^3$	Volume of a sphere
$A = \pi r\sqrt{r^2 + h^2}$	Lateral surface area of a right circular cone
$S_n = \frac{n}{2}[2a + (n - 1)d] = n\left(\frac{a + a_n}{2}\right)$	Sum of an arithmetic series
$S_n = \frac{a(1 - r^n)}{1 - r}$	Sum of a geometric series
$\sum_{n=0}^{\infty} ar^n = \frac{a}{1 - r}, \; \lvert r \rvert < 1$	Sum of an infinite geometric series
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Distance formula
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$	Midpoint formula
$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$	Slope
$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$	Law of sines
$c^2 = a^2 + b^2 - 2ab \cos C$	Law of cosines
$s^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n - 1}$	Variance
$s = r\theta$	Arc length
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Quadratic formula

PRACTICE QUESTIONS

Field 10: Mathematics

All examinees taking the Mathematics test (Field 10) will be provided with a scientific calculator with functions that include the following: addition, subtraction, multiplication, division, square root, percent, sine, cosine, tangent, exponents, and logarithms. See the current AEPA registration bulletin at www.aepa.nesinc.com for a list of available calculator models that may be provided on the day of the test. **You may NOT bring your own calculator to the test.**

1. A high school graduate recently started a new summer job. She worked 20% more hours in her third week on the job than she did in the second week. She worked 30% more hours in the second week than she did in the first week, and she worked 10% fewer hours in the first week than her regularly scheduled weekly hours. If the high school graduate worked 46 hours in her third week on the job, approximately what are her regularly scheduled weekly work hours?

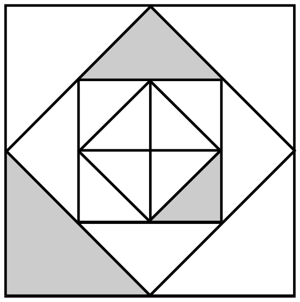
   A. 27

   B. 33

   C. 38

   D. 45

2. **Use the figure below to answer the question that follows.**



What fraction of the figure shown above is shaded?

A. $\frac{7}{32}$

B. $\frac{3}{16}$

C. $\frac{1}{4}$

D. $\frac{3}{8}$

3. **Use the problem below to answer the question that follows.**

> A factory's energy costs in January are $1500. The factory owner estimates that the cost of energy will decrease by 1.7% per month over the next six months. Based on this estimate, what will the factory's energy costs be in March of the same year?

Which of the following represents the solution to the problem above?

A. $1500 – ($1500 × 0.017 × 0.017)

B. $1500 × 0.983 × 0.983

C. ($1500 × 0.983) + ($1500 × 0.966)

D. ($1500 – ($1500 × 0.83)) × 0.83

4. If z_1 and z_2 are complex numbers with $z_1 = -1 + i\sqrt{3}$ and $z_2 = 2i$, then which of the following is the polar representation of the product $z_1 z_2$?

A. (2, 30°)

B. (2, 210°)

C. (4, 30°)

D. (4, 210°)

5. **Use the proof below to answer the question that follows.**

Prove that $-1a = -a$.

Proof:

1. $-1a + a = -1a + 1a$
2. $\quad = (-1 + 1)a$
3. $\quad = 0a$
4. $\quad = 0$
5. $\quad = a + (-a)$
6. $\quad = -a + a$
7. $\therefore -1a = -a$

Which number properties justify steps 2 and 6, respectively, in the above proof?

A. the additive inverse property and the associative property

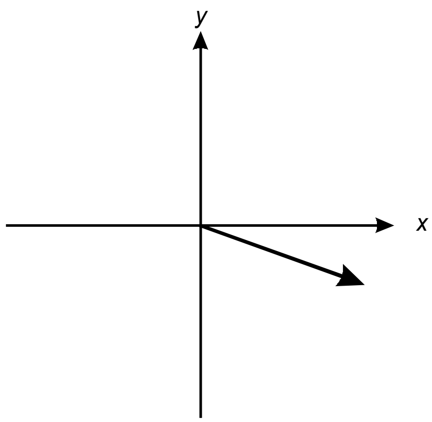
B. the distributive property and the commutative property

C. the additive inverse property and the commutative property

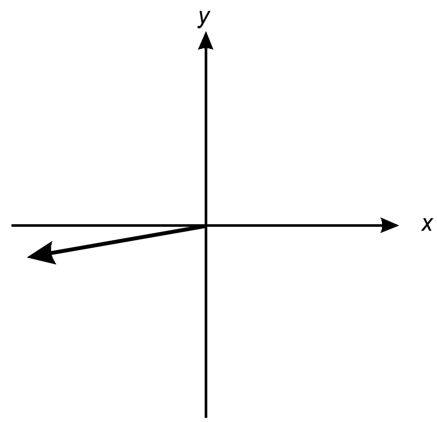
D. the distributive property and the associative property

6. Given that $z = \cos 10° + i \sin 10°$, which of the following represents z^2 in vector form?

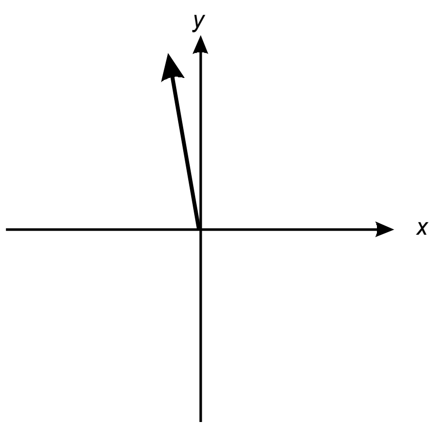
A.



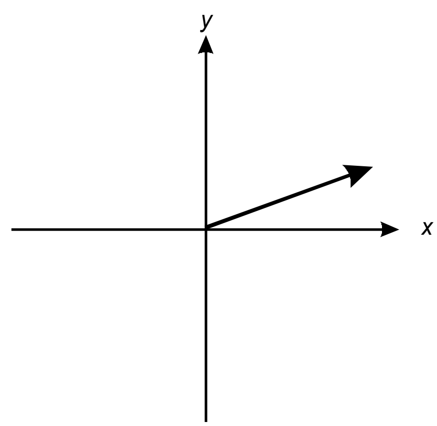
B.



C.



D.



7. If x is a positive number such that x mod3 = 1 and x mod8 = 5, which of the following is the lowest possible value of x?

A. 5

B. 11

C. 13

D. 17

8. The Euclidean algorithm is used to find the greatest common divisor of 2000 and 433. If the first step of the algorithm is expressed as $2000 = 4 \times 433 + 268$, then which of the following could represent the third step of the algorithm?

   A. $165 = 3 \times 53$

   B. $165 = 1 \times 103 + 62$

   C. $268 = 4 \times 67$

   D. $268 = 1 \times 165 + 103$

9. In how many ways may a person order an ice cream cone with scoops of three different flavors if there are ten flavors to choose from?

   A. $\frac{10!}{3!7!}$

   B. $\frac{10!}{7!}$

   C. $\frac{10!}{3!}$

   D. $\frac{10!}{3}$

10. On the first of March, some water is put into a water tank. For the next 32 days, 3 gallons of water is added to the tank each day. If, at the end of this period, there are 101 gallons of water in the tank, how many gallons were put in the tank on the first of March?

    A. 2

    B. 5

    C. 8

    D. 11

11. A photographer is trying to arrange four children in a row for a picture. Each child is wearing a different color sweater. If the child in the red sweater does not want to be placed on either end of the row and wants to be placed immediately to the right of the child in the blue sweater, in how many ways can the four children be arranged?

    A. 2

    B. 4

    C. 6

    D. 8

12. **Use the list of numbers below to answer the question that follows.**

    19, 5, 16, 12, *k*, 11, 3, 20

    If the mean of the list of numbers above is 12, what is the median of the list?

    A. 10

    B. 10.5

    C. 11

    D. 11.5

13. A list of various numbers has an average value of 24. If the number 24 is added to the list, which of the following describes the effect on the average and standard deviation of the original list?

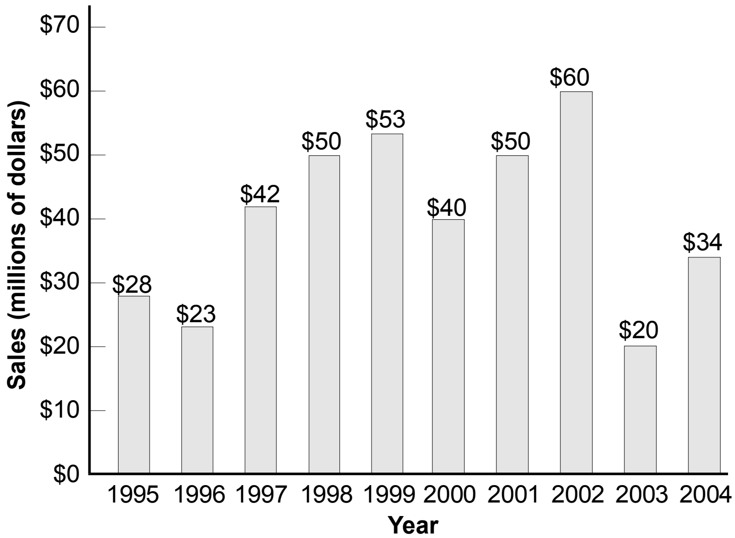
    A. The average and standard deviation remain the same.

    B. The average remains the same and the standard deviation decreases.

    C. The average and standard deviation decrease.

    D. The average decreases and the standard deviation remains the same.

Use the graph below to answer the two questions that follow.



14. The graph above represents a company's annual sales in millions of dollars. Which of the following statistical measures of the company's annual sales is equal to $50 million?

    A. median

    B. range

    C. mode

    D. standard deviation

15. Which of the following is closest to the 30th percentile of the company's annual sales?

    A. $28 million

    B. $42 million

    C. $50 million

    D. $60 million

16. Six individual socks are drawn at random without replacement from a basket containing seven different pairs of socks. If the six socks drawn are all different from one another, what is the probability that the next sock drawn at random from the basket is also different from the rest?

A. $\frac{1}{14}$

B. $\frac{1}{8}$

C. $\frac{1}{7}$

D. $\frac{1}{4}$

17. A point (x, y) is randomly chosen such that $0 \leq x \leq 1$ and $0 \leq y \leq 1$. What is the probability that $y \geq 2x$?

A. $\frac{1}{8}$

B. $\frac{1}{6}$

C. $\frac{1}{4}$

D. $\frac{1}{2}$

18. Of all the cars sold at a car dealership, 80% are not convertibles, 40% are red, and 5% are red convertibles. Given that a car sold by a salesperson at this dealership is a convertible, what is the probability that it is red?

    A. 0.08

    B. 0.25

    C. 0.32

    D. 0.40

19. The number of peanuts in a six-ounce bag of peanuts is normally distributed with a mean of 230 and a standard deviation of 20. What is the probability that a randomly selected six-ounce bag of peanuts has more than 260 peanuts in it?

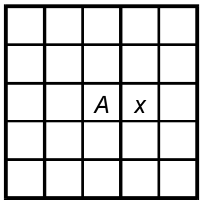
    A. 0.0668

    B. 0.2166

    C. 0.4332

    D. 0.9332

20. **Use the diagram below to answer the question that follows.**



An ant is placed on the square marked *A* on the grid above. If the ant can move one square at a time in any direction, including diagonally, what is the probability that after two moves the ant is on the square labeled *x*?

A. $\frac{1}{32}$

B. $\frac{1}{16}$

C. $\frac{1}{8}$

D. $\frac{1}{4}$

21. For each subject in a study, researchers record values for random variables representing age, number of years of education, income level, and length of time with current employer. The researchers want to determine whether there is a correlation between any of the two variables. Which of the following graphical representations would be most effective in enabling the researchers to make this determination?

A. scatter plots

B. graphs of frequency distributions

C. box and whisker plots

D. graphs of density functions

22. A television program asked its viewers to respond to the question, "Do you think the voting age should be raised to 21?" Of the 550 people who responded, 21% responded "yes," 72% responded "no," and 7% responded "unsure." Which of the following best explains why the results of this poll should not be used to make inferences about the entire population in the country?

   A. The question posed in the poll is inherently biased.

   B. The sample size is not large enough for any resulting claim to be valid.

   C. The "unsure" responses make the findings inaccurate.

   D. The respondents to the poll do not constitute a representative sample of the whole population.

23. The inverse of which of the following functions is a function?

   A. $y = 2x - 1$

   B. $y = x^2 + x - 4$

   C. $y = | x - 3 |$

   D. $y = \sqrt{16 - x^2}$

24. Consider the graphs of the functions f(x) = $ax + b$ and g(x) = $hx + k$, where a and b are negative integers and $h = \frac{1}{2}a$ and $k = b$. Which of the following describes a difference between the graphs of f(x) and g(x)?

   A. The x-intercept of the graph of g(x) is half the x-intercept of the graph of f(x).

   B. The x-intercept of the graph of g(x) is double the x-intercept of the graph of f(x).

   C. The y-intercept of the graph of g(x) is half the y-intercept of the graph of f(x).

   D. The y-intercept of the graph of g(x) is double the y-intercept of the graph of f(x).

25. What is the domain of p(x) = $\frac{x^2 - 5x - 14}{x^2 - x - 6}$?

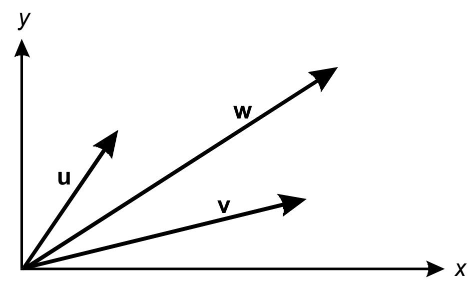
   A. $(-\infty, -2) \cup (3, \infty)$

   B. $(-\infty, 3) \cup (3, \infty)$

   C. $(-\infty, 2) \cup (2, -3) \cup (-3, \infty)$

   D. $(-\infty, -2) \cup (-2, 3) \cup (3, \infty)$

26. **Use the diagram below to answer the question that follows.**



In the diagram above, what is the length of vector **w** = **u** + **v** if **u** = (a, b) and **v** = (c, d)?

A. $\sqrt{(a + c)^2 + (b + d)^2}$

B. $\sqrt{(a - c)^2 + (b - d)^2}$

C. $\sqrt{(a + c)^2 - (b + d)^2}$

D. $\sqrt{(a - c)^2 - (b - d)^2}$

27. **Use the problem below to answer the question that follows.**

> A rectangular box has length *L*, width *W*, and height *H*. The sum of its length, width, and height is 75 cm, its length is twice the sum of its width and height, and its height is 5 cm less than twice its width. What are the dimensions of this box?

If the problem above were to be solved using matrices, an appropriate first step would be to find which of the following?

A. the determinant of $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 2 & 1 \end{bmatrix}$

B. the product $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 2 & 1 \end{bmatrix}\begin{bmatrix} 75 \\ 0 \\ -5 \end{bmatrix}$

C. the inverse of $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & -2 \\ 0 & -2 & 1 \end{bmatrix}$

D. the product $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & -2 \\ 0 & -2 & 1 \end{bmatrix}\begin{bmatrix} L & W & H \end{bmatrix}$

28. **Use the matrix below to answer the question that follows.**

$$M = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

The third row of matrix M above is multiplied by −6 and then added to its second row. In terms of the determinant of M, what is the determinant of the resulting matrix?

A. −6 times the determinant of M

B. the same as the determinant of M

C. 12 times the determinant of M

D. the negative of the determinant of M

29. The equation $2x^2 - 4x + m = 0$ has a real number solution if which of the following is true?

    A. $m \geq -2$

    B. $m \geq 2$

    C. $m \leq -2$

    D. $m \leq 2$

30. The population in town A declined at a constant rate from 10,000 in the year 1990 to 9,040 in the year 1998. The population in town B increased at a constant rate from 4,000 in the year 1994 to 4,560 in the year 1998. If the rates of change of population in towns A and B remain the same, in approximately what year will the populations in the two towns be equal?

    A. 2007

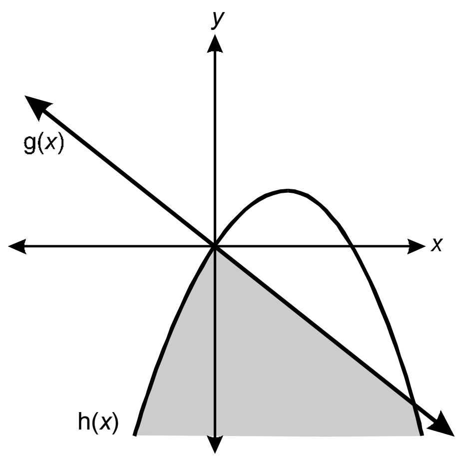
    B. 2013

    C. 2015

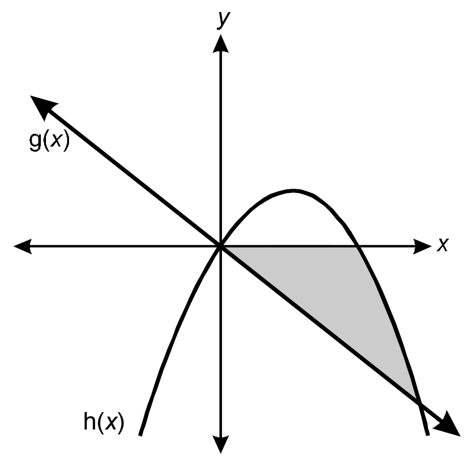
    D. 2021

31. Which of the following represents the solution set for the system of inequalities $g(x) \leq -2x$ and $h(x) \leq 8x - x^2$?

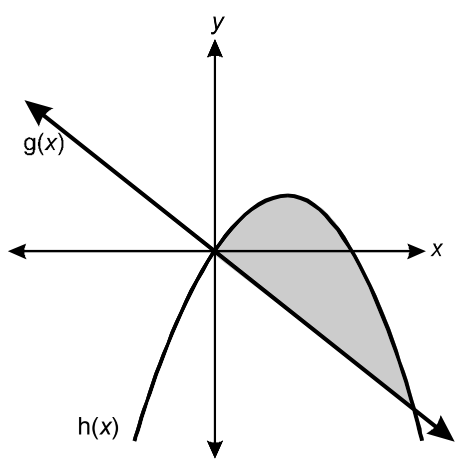
A.



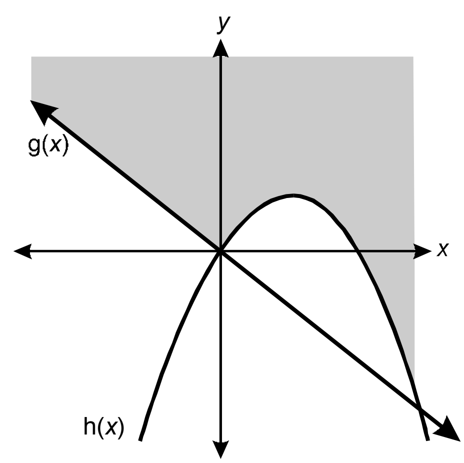
B.



C.



D.



32. **Use the information below to answer the question that follows.**

Time (in minutes)	Speed (in knots)
0	20
1	10
2	5
3	2.5
4	1.25

Researchers are running an experiment to determine the effects of the force of water on a boat. The experiment involves bringing the boat to a speed of 20 knots and then turning the motor off. The table above shows the speed of the boat at given times. If s_0 denotes the speed of the boat at the moment when the motor is turned off, which of the following functions describes the speed of the boat at any time t?

A. $s(t) = s_0 2^{-t}$

B. $s(t) = s_0 e^{-2t}$

C. $s(t) = s_0 t^{-2}$

D. $s(t) = s_0 - \frac{1}{2} s_0 t$

33. The mass of a bacteria colony at time t is given by $B(t) = ke^{rt}$, where r is the rate of growth of the mass of bacteria. If the initial mass of the bacteria colony is 500 units, which of the following represents the growth rate of the mass as a function of t and $B(t)$?

A. $r = \dfrac{\ln 500 - \ln k}{t}$

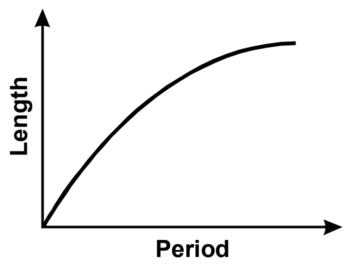
B. $r = \dfrac{\ln B(t)}{t \ln 500}$

C. $r = \dfrac{\ln B(t) - \ln 500}{t}$

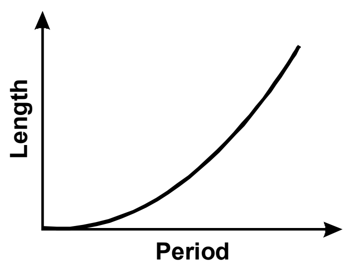
D. $r = \dfrac{\ln(B(t) - 500)}{t \ln k}$

34. The period of a pendulum, defined as the length of time the pendulum takes to make one complete swing, varies directly with the square root of the length of the pendulum. Which of the following graphs represents the length of a pendulum as a function of its period?

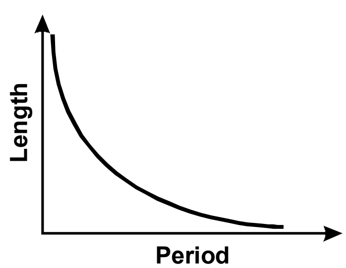
A.



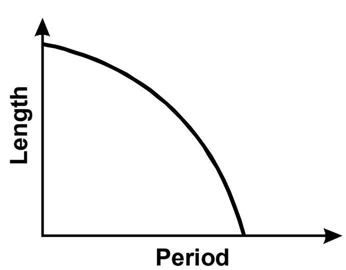
B.



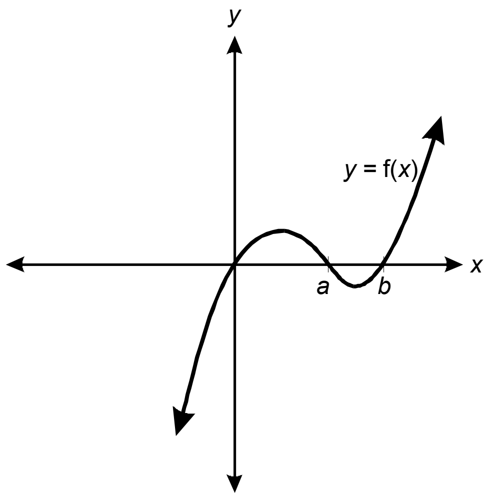
C.



D.



35. **Use the graph below to answer the question that follows.**



Given that the axes in the graph above intersect at the origin, which of the following represents the function f(x)?

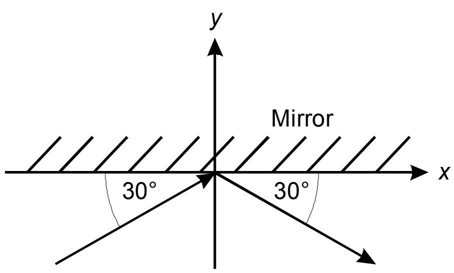
A. $f(x) = x^2 - ax - bx + ab$

B. $f(x) = x^2 + ax + bx + ab$

C. $f(x) = x^3 - ax^2 - bx^2 + abx$

D. $f(x) = x^3 + ax^2 + bx^2 + abx$

36. **Use the diagram below to answer the question that follows.**



A mirror is placed on the horizontal axis of a coordinate system. A laser beam is shone onto the mirror at the origin such that the angles of incidence and reflection are 60°, as shown above. Which of the following functions represents the trajectory of the laser beam?

A. $y = -\frac{\sqrt{3}}{3}\,|x|$

B. $y = -\frac{1}{2}\,|x|$

C. $y = -\sqrt{3}\,|x|$

D. $y = -2\,|x|$

37. A citrus grower finds that when there are 100 orange trees planted in his field, the yield per tree is 450 oranges. He estimates that the yield will decrease by 3 oranges per tree for each additional tree planted in the same field. If the total yield of the citrus grower's field is given by a function of the form $y(x) = ax^2 + bx + 45{,}000$, where x denotes each additional tree planted, what is the value of b?

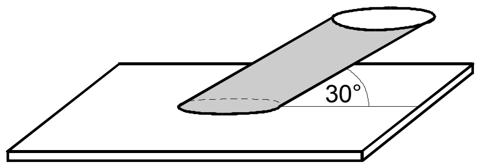
A. 150

B. 300

C. 450

D. 550

38. **Use the diagram below of a tilted circular cylinder to answer the question that follows.**



If the circular cylinder above has a radius of 3 cm and a slant height of 12 cm, what is the volume of the cylinder?

A. $54\pi\ \text{cm}^3$

B. $54\sqrt{3}\ \pi\ \text{cm}^3$

C. $108\pi\ \text{cm}^3$

D. $108\sqrt{3}\ \pi\ \text{cm}^3$

39. Each wheel on a vehicle has a radius of 20 inches. If the vehicle's wheels rotate at 340 revolutions per minute, what is the approximate speed of the vehicle in miles per hour?

A. 20 miles per hour

B. 30 miles per hour

C. 40 miles per hour

D. 50 miles per hour

40. A homeowner is planning to paint an 8 foot by 22 foot wall and wants to determine the cost of the project. She tests a 14 inch by 18 inch section of the wall and finds that 1 ounce of paint is needed for good coverage. If she uses paint that costs $28 per gallon, what will be the cost of the paint needed to give the entire wall a good coverage of paint?

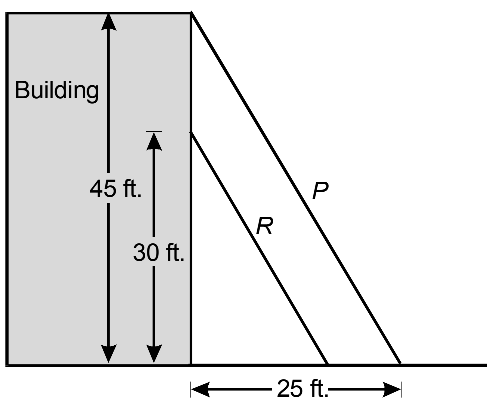
A. $17.82

B. $22.00

C. $28.16

D. $33.69

41. **Use the diagram below to answer the question that follows.**



Ladder *P* shown above touches the top of a 45-foot high building with its base placed 25 feet from the base of the building. If ladder *R* is placed parallel to ladder *P* such that it touches the building 30 feet above the ground, what is the length of ladder *R*?

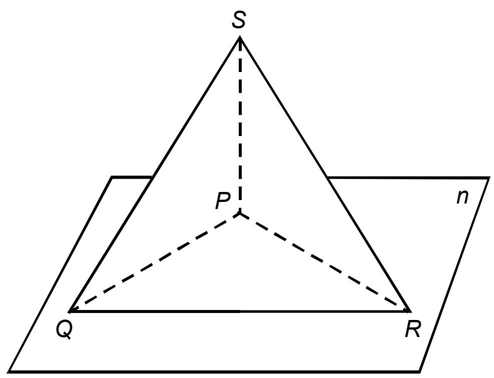
A. 16.67 feet

B. 20.00 feet

C. 34.32 feet

D. 41.48 feet

42. **Use the diagram below to answer the question that follows.**



Given: $\triangle$ *PQR* in plane *n*;
point S not in plane *n*;
$\angle PQR \cong \angle PRQ$;
$\angle SQR \cong \angle SRQ$.

Which of the following congruence relationships should be used to prove $\triangle SPQ \cong \triangle SPR$ in the diagram above?

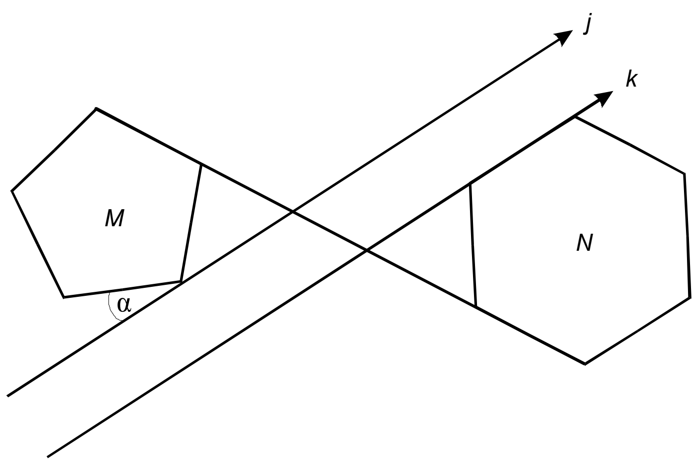
A. SSS

B. SAS

C. ASA

D. SAA

43. **Use the diagram below to answer the question that follows.**



In the diagram above, *M* and *N* are regular polygons and lines *j* and *k* are parallel. What is the value of $m\angle\alpha$?

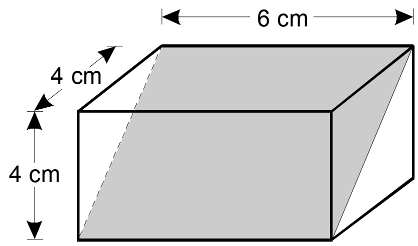
A. 14°

B. 24°

C. 36°

D. 40°

44. **Use the diagram below to answer the question that follows.**



A rectangular solid is intersected by a plane as shown above. What is the area of the cross section formed by the intersection?

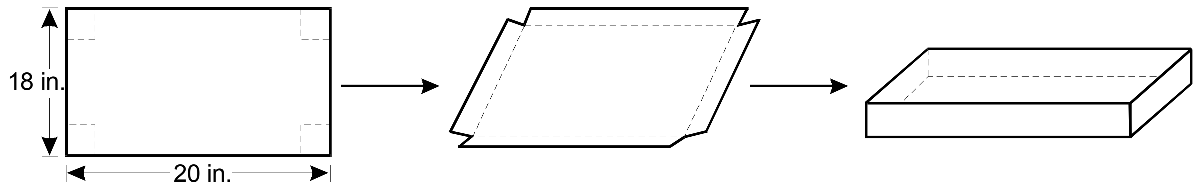
A. $16\sqrt{2}$ cm^2

B. $24\sqrt{2}$ cm^2

C. 36 cm^2

D. 48 cm^2

45. **Use the diagram below to answer the question that follows.**



A piece of cardboard measuring 18 inches by 20 inches is to be made into a box by cutting a square out of each corner of the piece of cardboard and then folding up the remaining flaps as shown above. If each square cut out of the corners has sides of length x inches, what is the volume of the resulting box?

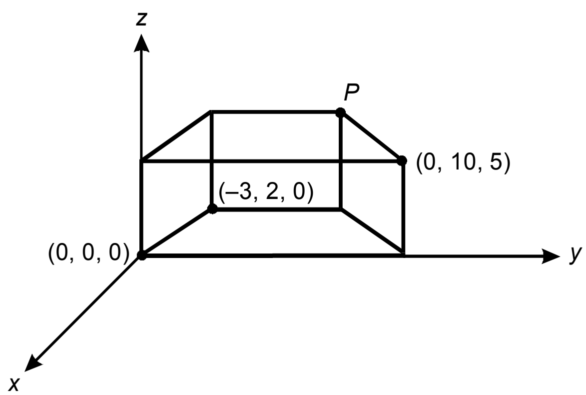
A. $x^3 - 38x^2 + 360x$ in.^{3}

B. $2x^3 - 58x^2 + 360x$ in.^{3}

C. $2x^3 - 56x^2 + 360x$ in.^{3}

D. $4x^3 - 76x^2 + 360x$ in.^{3}

46. **Use the diagram below to answer the question that follows.**



Given that the figure above is a right prism with an isosceles trapezoidal base, what are the coordinates of point *P*?

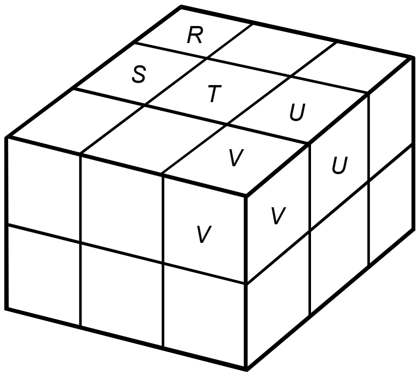
A. (–5, 8, 5)

B. (–5, 10, 5)

C. (–3, 8, 5)

D. (–3, 10, 5)

47. **Use the diagram below to answer the question that follows.**



The rectangular structure above is composed of 18 cubes, each of which has sides of length one unit. Of the five cubes labeled *R*, *S*, *T*, *U*, and *V*, cube *V* and one other cube are removed from the structure. If the surface area of the remaining structure, including the bottom, is 46 square units, which of the labeled cubes other than *V* was removed?

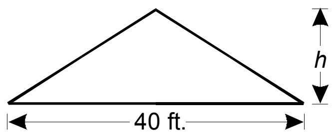
A. *R*

B. *S*

C. *T*

D. *U*

48. **Use the diagram below to answer the question that follows.**



The cross section of the roof of a building shown above has a width of 40 feet and a slope of $\frac{4}{5}$ feet. If the cross section of the roof represents an isosceles triangle, what is the height *h* of the roof?

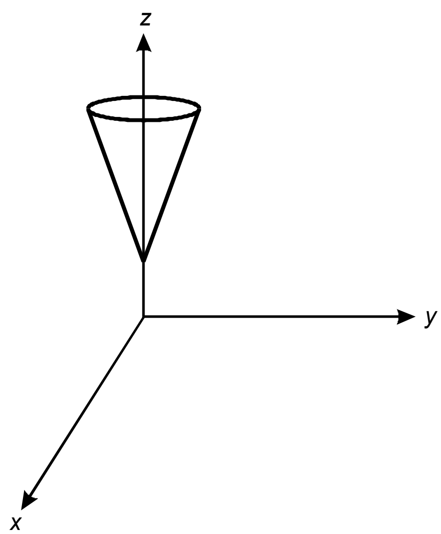
A. 16 feet

B. 25 feet

C. 32 feet

D. 50 feet

49. **Use the diagram below to answer the question that follows.**



When the cone above is intersected by a plane, the resulting curve could be represented by which of the following equations?

A. $y = x^3$

B. $x^2 + y^2 = 16$

C. $y = \sqrt{x^2 - x}$

D. $\frac{x^2}{9} - \frac{y^2}{25} = 1$

50. A triangle *RST* has coordinates (0,2), (6, –4), and (–4,–12). Under a dilation centered at point *R*, with a scale factor of $\frac{1}{2}$, triangle *RST* becomes triangle *R′S′T′*. Which of the following is the equation of the line joining points S′ and *T′*?

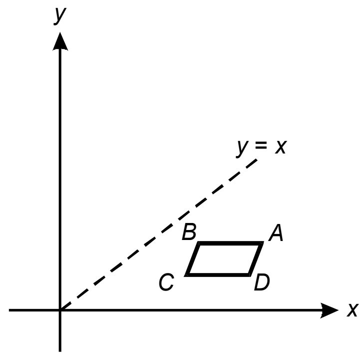
A. $y = \frac{4}{5}x - \frac{17}{5}$

B. $y = \frac{4}{5}x - \frac{27}{5}$

C. $y = \frac{5}{4}x - \frac{19}{4}$

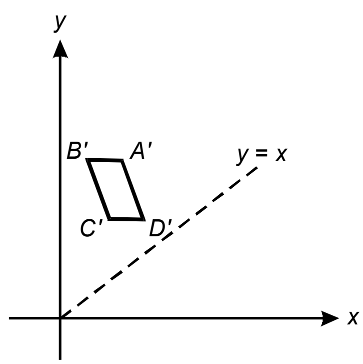
D. $y = \frac{5}{4}x - \frac{29}{4}$

51. **Use the diagram below to answer the question that follows.**

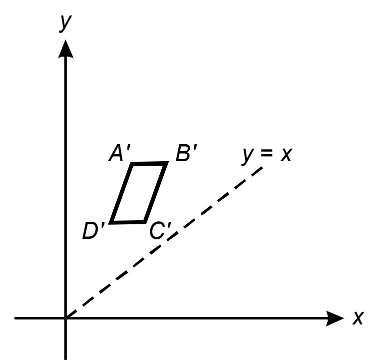


The parallelogram *ABCD* above is rotated 90° clockwise around point *D* and then reflected over the line *y* = *x*, resulting in the parallelogram *A´B´C´D´*. Which of the following represents *A´B´C´D´*?

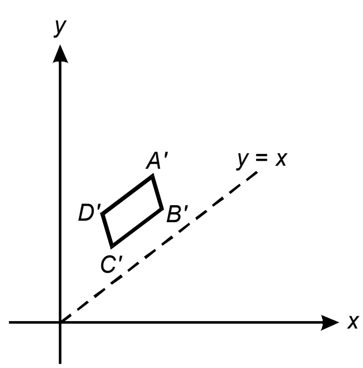
A.



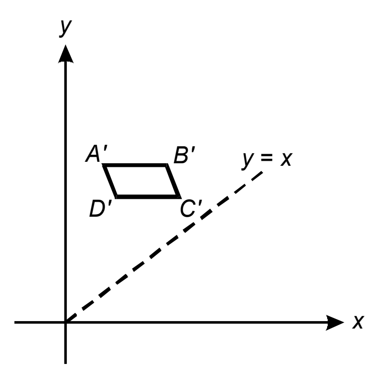
B.



C.



D.



52. The graph of the parabola $y = x^2 - 1$ is translated 3 units to the right and then reflected about the y-axis. What are the coordinates of the vertex of the parabola that result under this transformation?

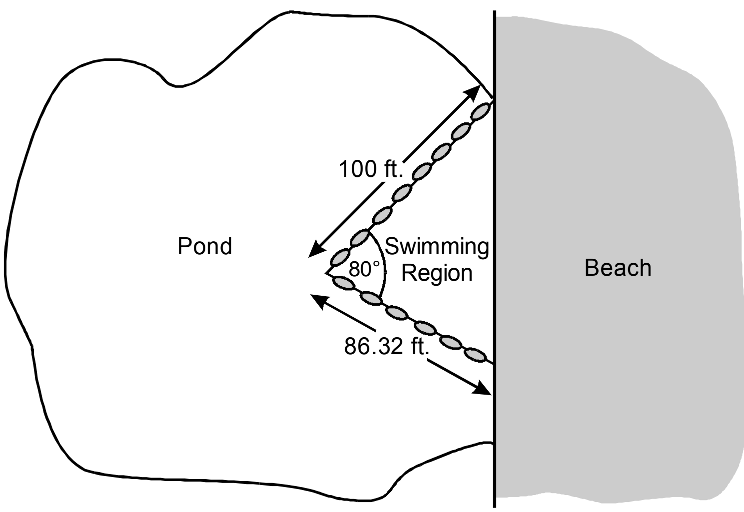
A. (3, –1)

B. (–3, –1)

C. (3, 1)

D. (–3, 1)

53. **Use the diagram below to answer the question that follows.**



To provide a swimming area for local residents, a town has enclosed a section of a pond with two sets of strings of buoys of length 86.32 feet and 100 feet. The strings of buoys form a triangular region with the beach on the side of the pond, as shown above. What is the area of the swimming region?

A. 2500.00 square feet

B. 3740.98 square feet

C. 4250.43 square feet

D. 6011.14 square feet

54. Given that $\sec \theta = \frac{5}{3}$ and $0 \leq \theta \leq \frac{\pi}{2}$, what is the value of the expression $\tan \theta + \frac{\sin^2 \theta}{\cos \theta}$?

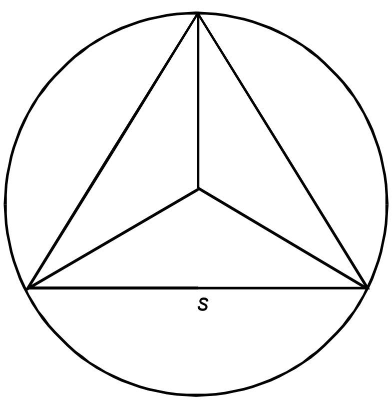
A. $\frac{27}{20}$

B. $\frac{109}{60}$

C. $\frac{29}{15}$

D. $\frac{12}{5}$

55. **Use the diagram below to answer the question that follows.**



An equilateral triangle with side length *s* is inscribed in a circle, as shown above. What is the circumference of the circle?

A. $\frac{\sqrt{3}}{3}\pi s$

B. $\frac{2\sqrt{3}}{3}\pi s$

C. $\sqrt{3}\pi s$

D. $\frac{4\sqrt{3}}{3}\pi s$

56. How many solutions does the equation $2 \cos^2 x = 1$ have on the interval $0 \le x < 2\pi$?

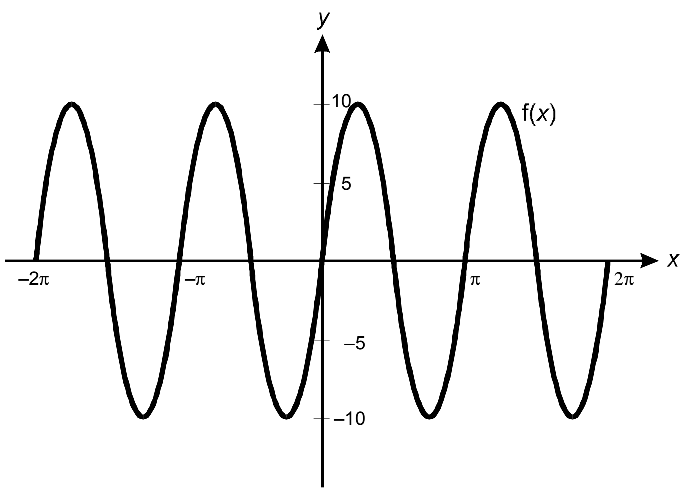
A. 1

B. 2

C. 3

D. 4

57. **Use the graph of f(x) below to answer the question that follows.**



Which of the following functions has a graph that looks like this graph of f(x) shifted $\frac{\pi}{2}$ units to the right and has a period that is two times the period of f(x)?

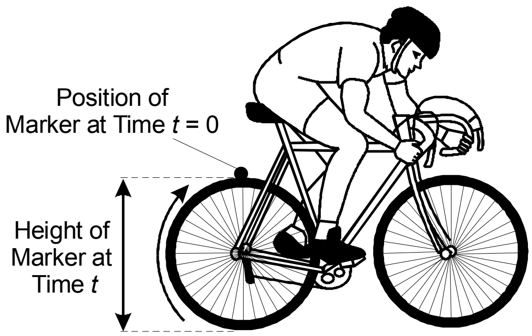
A. $10 \sin \left(x + \frac{\pi}{2}\right)$

B. $10 \sin \left(x - \frac{\pi}{2}\right)$

C. $10 \sin \left(2x + \frac{\pi}{2}\right)$

D. $10 \sin \left(2x - \frac{\pi}{2}\right)$

58. **Use the diagram below to answer the question that follows.**



The wheels on an athlete's bicycle each have a diameter of 60 centimeters. At time $t = 0$, a marker is placed at the top edge of the rear wheel of the bicycle, as shown above. If the athlete rides the bicycle at a speed of 30π centimeters per second, which of the following represents the height h of the marker at time t?

A. $h(t) = 60 \cos(\pi t)$

B. $h(t) = 60 \cos(t)$

C. $h(t) = 30 \cos(\pi t) + 30$

D. $h(t) = 30 \cos(t) + 30$

59. What is the value of $\lim\limits_{k \to 0} \dfrac{k^2 + 2k}{k}$?

A. 0

B. 1

C. 2

D. undefined

60. What is the value of $\sum\limits_{n=1}^{\infty} \left(-\frac{4}{5}\right)^{3n}$?

A. $-\frac{4}{9}$

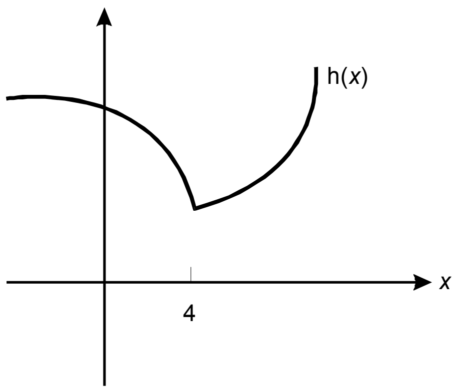
B. $-\frac{64}{189}$

C. $\frac{5}{9}$

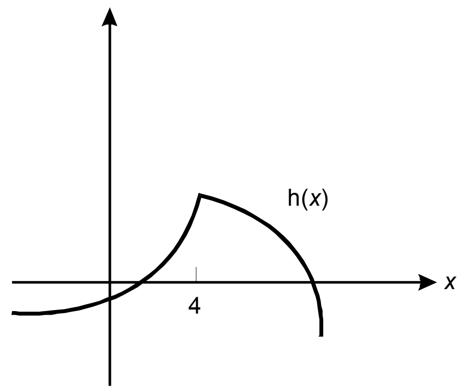
D. $\frac{125}{189}$

61. A function h(x) has a first derivative of zero at $x = 4$, is concave up on the interval $(-\infty, 4)$, and is concave down on the interval $(4, \infty)$. Which of the following best represents the graph of h(x)?

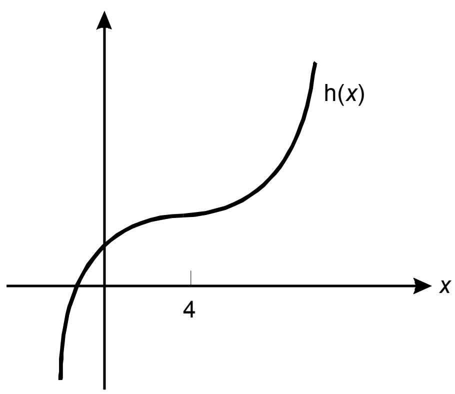
A.



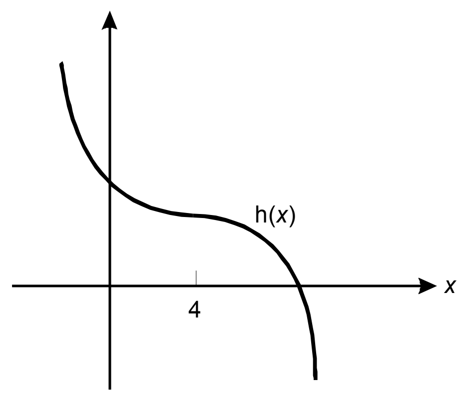
B.



C.



D.



62. What is the local minimum value of $f(x) = 2x^3 - 3x^2 - 12x$?

A. –20

B. –13

C. –4

D. 7

63. The daily amount of smog produced by cars in a city is given by $s = 3 + 4x^{3/2}$, where s is the smog concentration in parts per million and x is the number of cars in the city. If the number of cars in the city increases at a rate of 20 per day, at what rate will the smog concentration change when there are 2,500 cars in the city?

A. 300 parts per million per day

B. 6,000 parts per million per day

C. 10,000 parts per million per day

D. 500,000 parts per million per day

64. What is the area of the region bounded by the graphs of $y = x^2$, $y = x$, and $x = 2$?

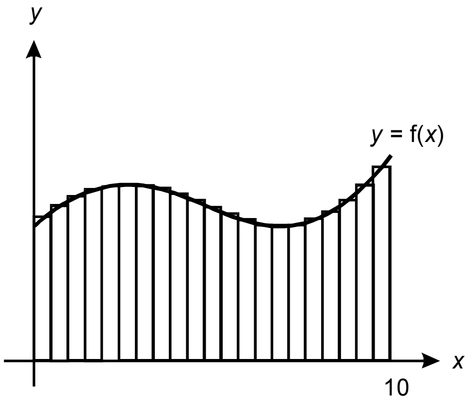
A. $\frac{1}{6}$

B. $\frac{2}{3}$

C. $\frac{5}{6}$

D. $\frac{23}{6}$

65. **Use the graph below to answer the question that follows.**



The interval [0,10] above is divided into n subintervals over which rectangles are constructed. The height and width of each of the n rectangles are given by $f(x_i)$ and $\frac{10}{n}$, respectively, where $1 \le i \le n$. Using Riemann sums, $\int_0^{10} f(x)\,dx$ is equal to which of the following?

A. $\lim\limits_{n \to \infty} \sum\limits_{i=1}^{n} f(x_i) \frac{10}{n}$

B. $\lim\limits_{dx \to \infty} \sum\limits_{i=1}^{n} f(x_i)\,dx$

C. $\lim\limits_{n \to 0} \sum\limits_{i=1}^{n} f(x_i) \frac{10}{n}$

D. $\lim\limits_{dx \to 0} \sum\limits_{i=1}^{n} f(x_i) \frac{10}{n}\,dx$

66. The number of journalists in a metropolitan region is increasing at a rate of $18t^2 + 60t$ journalists per year. If $t = 0$ represents the present, what will be the increase in the total number of journalists in the region over the next three years?

A. 168

B. 258

C. 342

D. 432

67. A teacher wants to instruct her students on appropriate and efficient techniques for solving mathematics problems. Which of the following problems would be most appropriate for her to suggest that they solve by making a Venn diagram?

A. What is the least common multiple of the integers 12, 26, and 35?

B. Out of a group of 8 people, how many different committees of 3 can be formed?

C. What are the local maxima of the function $y = 2x^4 + 3x^3 - 5x - 10$?

D. Out of 26 students, 13 play football, 9 play tennis, and 4 play both. How many of the students play neither sport?

68. A building contractor has a rectangle with sides of integer lengths p and q that is to be covered using the smallest number of identical square tiles possible. The tiles are to be placed adjacent to one another and should not exceed the area of the rectangle. Which of the following represents the side length of the tile that should be used?

A. the difference between p and q

B. the remainder of $p \div q$

C. the smallest number that is divisible by both p and q

D. the largest number that divides both p and q

69. Which of the following would be the most appropriate method to use to find the number of points of intersection of the functions $\cot(x)$ and $\sin(x) - 1$ over the interval $[0, \frac{3\pi}{2}]$?

A. Find the zeros of the equation $\cot(x) - \sin(x) + 1 = 0$.

B. Determine where the graph of the function $\cot(x) - \sin(x) + 1$ crosses the y-axis.

C. Solve the equation $\cot(x) = \sin(x) - 1$ algebraically.

D. Graph the functions $\cot(x)$ and $\sin(x) - 1$ on the same coordinate plane.

70. **Use the statement below to answer the question that follows.**

> Of the students in a high school, one fifth are freshmen, one fourth are sophomores, one third are juniors, and the remaining 150 students are seniors.

If N denotes the total number of students in the school, which of the following equations represents the statement above?

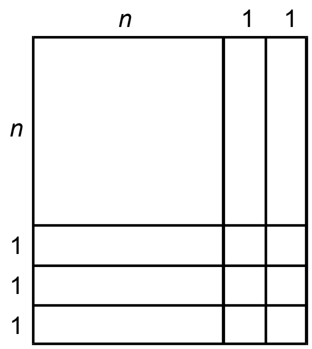
A. $\frac{1}{5}N + \frac{1}{4}N + \frac{1}{3}N = 150$

B. $\frac{1}{5}N + \frac{1}{4}N + \frac{1}{3}N + N = 150$

C. $\frac{1}{5}N + \frac{1}{4}N + \frac{1}{3}N + 150 = N$

D. $\frac{1}{5}N + \frac{1}{4}N + \frac{1}{3}N + 150N = N$

71. **Use the diagram below to answer the question that follows.**



Which of the following concepts can be illustrated most effectively using the diagram above?

A. permutations and combinations

B. multiplication of polynomials

C. greatest common factors

D. prime factorization of numbers

72. If A and B are two finite sets in the universal set, which of the following is equivalent to $B - (A \cap B)$?

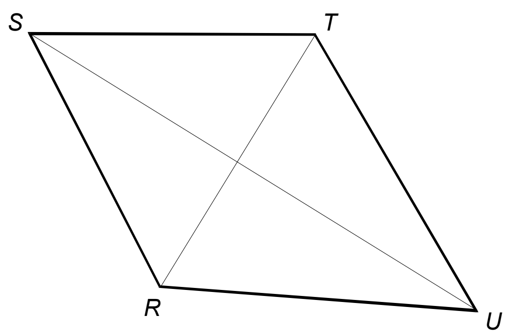
A. $(A \cup B) \cap B^c$

B. $(A \cup B)^c \cup B$

C. $(A \cap B) \cup B^c$

D. $(A \cap B)^c \cap B$

73. **Use the diagram below to answer the question that follows.**



In quadrilateral *RSTU* above, $m\angle TUS = 28°$ and $m\angle RSU = 31°$. To construct an indirect proof that quadrilateral *RSTU* is not a parallelogram, the first step would be to assume that:

A. *RSTU* is not a quadrilateral.

B. *RSTU* is a parallelogram.

C. $\overline{RT}$ and $\overline{SU}$ are not perpendicular.

D. $\overline{RT}$ and $\overline{SU}$ are congruent.

74. **Use the statement below to answer the question that follows.**

If a triangle is not isosceles, then it is not equilateral.

If the statement above is true, then which of the following statements must also be true?

A. If a triangle is isosceles, then it is equilateral.

B. If a triangle is not equilateral, then it is not isosceles.

C. If a triangle is equilateral, then it is isosceles.

D. If a triangle is not equilateral, then it is isosceles.

75. **Use the statement below to answer the question that follows.**

If $a > b$, then $\frac{1}{a} < \frac{1}{b}$.

Which of the following would be a counterexample to the statement above?

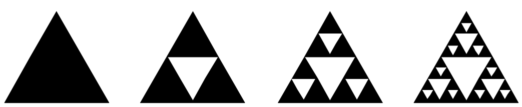
A. Let $0 < a < 1$ and $0 < b < 1$.

B. Let a be a positive integer and let $0 < b < 1$.

C. Let a and b be negative integers.

D. Let a be a positive integer and let b be a negative integer.

76. **Use the figures below to answer the question that follows.**



Which of the following could be used to determine the total shaded area of the figures shown above?

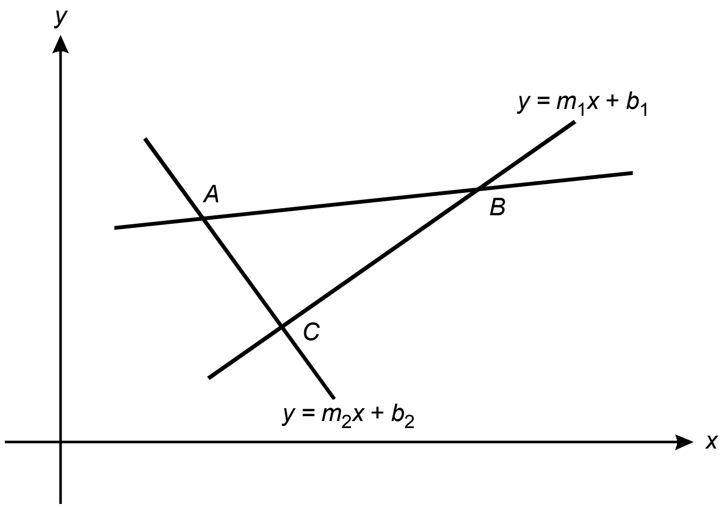
A. tessellations

B. geometric series

C. congruence

D. recursive functions

77. **Use the diagram below to answer the question that follows.**



Under which of the following conditions is it possible to conclude that triangle *ABC* shown above is a right triangle?

A. $m_1 = m_2$

B. $m_1 = -m_2$

C. $m_1 m_2 = 1$

D. $m_1 m_2 = -1$

78. Students in an algebra class are comparing the graphs of $f(x) = x + a$ and $g(x) = -x - a$. This topic of study is related to which of the following concepts from transformational geometry?

A. dilation

B. reflection

C. rotation

D. translation

79. In spherical geometry, lines are defined as "great circles" that span the circumference of a sphere. Which of the following statements from Euclidean geometry holds in spherical geometry?

    A. The sum of the interior angles of a triangle is 180°.

    B. The square of the hypotenuse of a right triangle equals the sum of the squares of the other two sides.

    C. Given any line, there exists at least one line parallel to it.

    D. Given any two points, at least one line can be drawn through the points.

80. Which of the following best describes a corollary?

    A. A statement that follows readily from a theorem.

    B. A statement whose converse is also true.

    C. A statement that is accepted as true as the basis for argument.

    D. A statement used to supplement the proof of a proposition.

Below are the directions for the Mathematics performance assignment.

DIRECTIONS FOR THE PERFORMANCE ASSIGNMENT

This section of the test consists of a performance assignment. **The assignment can be found on the next page.** You are asked to prepare a written response of approximately 2–3 pages on the assigned topic. You should use your time to plan, write, review, and edit your response for the assignment.

Read the assignment carefully before you begin to work. Think about how you will organize your response. You may use any blank space in this test booklet to make notes, write an outline, or otherwise prepare your response. **However, your score will be based solely on the version of your response written in Written Response Booklet B.**

As a whole, your response must demonstrate an understanding of the knowledge and skills of the field. In your response to the assignment, you are expected to demonstrate the depth of your understanding of the content area through your ability to apply your knowledge and skills rather than merely to recite factual information.

Your response will be evaluated based on the following criteria.

- **PURPOSE:** the extent to which the response achieves the purpose of the assignment
- **SUBJECT MATTER KNOWLEDGE:** accuracy and appropriateness in the application of subject matter knowledge
- **SUPPORT:** quality and relevance of supporting details
- **RATIONALE:** soundness of argument and degree of understanding of the subject matter

The performance assignment is intended to assess subject knowledge content and skills, not writing ability. However, your response must be communicated clearly enough to permit scorers to make a valid evaluation of your response according to the criteria listed above. Your response should be written for an audience of educators in this field. The final version of your response should conform to the conventions of edited American English. This should be your original work, written in your own words, and not copied or paraphrased from some other work.

Be sure to write about the assigned topic. Please write legibly. You may not use any reference materials during the test. Remember to review your work and make any changes you think will improve your response.

Below is the scoring scale for the Mathematics performance assignment.

SUBJECT TESTS—PERFORMANCE ASSIGNMENT SCORING SCALE

Score Point	Score Point Description
4	**The "4" response reflects a thorough knowledge and understanding of the subject matter.** • The purpose of the assignment is fully achieved. • There is a substantial, accurate, and appropriate application of subject matter knowledge. • The supporting evidence is sound; there are high-quality, relevant examples. • The response reflects an ably reasoned, comprehensive understanding of the topic.
3	**The "3" response reflects an adequate knowledge and understanding of the subject matter.** • The purpose of the assignment is largely achieved. • There is a generally accurate and appropriate application of subject matter knowledge. • The supporting evidence is adequate; there are some acceptable, relevant examples. • The response reflects an adequately reasoned understanding of the topic.
2	**The "2" response reflects a limited knowledge and understanding of the subject matter.** • The purpose of the assignment is partially achieved. • There is a limited, possibly inaccurate or inappropriate, application of subject matter knowledge. • The supporting evidence is limited; there are few relevant examples. • The response reflects a limited, poorly reasoned understanding of the topic.
1	**The "1" response reflects a weak knowledge and understanding of the subject matter.** • The purpose of the assignment is not achieved. • There is little or no appropriate or accurate application of subject matter knowledge. • The supporting evidence, if present, is weak; there are few or no relevant examples. • The response reflects little or no reasoning about or understanding of the topic.

U	**The response is unrelated to the assigned topic, illegible, primarily in a language other than English, not of sufficient length to score, or merely a repetition of the assignment.**
B	**There is no response to the assignment.**

Practice Performance Assignment

81. **Read the information below; then complete the exercise that follows.**

A shipping service will accept a package for shipment only if the sum of the height and the girth (the perimeter of the base) is less than or equal to 108 inches. A company is planning to ship a package that has the shape of a right circular cylinder.

Using your knowledge of mathematical modeling and calculus, write an essay in which you:

- graph the set of all possible values of h in terms of r, where h represents the height of the cylinder and r represents the radius, and describe the characteristics of the graph (e.g., quadratic inequality with local maximum at [*a*, *b*], linear equation with slope m);
- derive an expression for the volume of the cylinder in terms of r, graph the expression over the domain of all possible values of r, and describe the characteristics of the graph; and
- find the dimensions of the cylinder of maximum volume that the shipping service will accept for shipment.

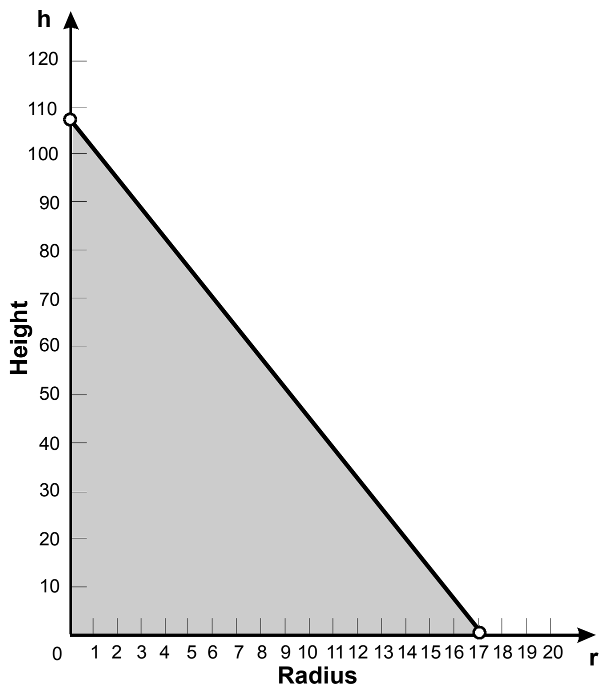
Be sure to show your work and explain the steps you used to find your answers.

Sample Performance Assignment Response: Score Point 4

Since the height and girth, or the perimeter of the base of the cylinder, must be less than or equal to 108 inches, we can write the following equation:

$h + 2\pi r \leq 108$

This is a linear inequality. To graph the inequality, note that the line $h + 2\pi r = 108$ is the border of the set of all possible values of h and r. This line is equivalent to $h = {}^{-}2\pi r + 108$, which is a linear function. The line has slope ${}^{-}2\pi = {}^{-}6.28$ and h-intercept 108. The r-intercept is $\frac{108}{2\pi}$ or about 17.2. This means that $0 < r < \frac{54}{\pi}$. The values $h = 0$ and $r = 0$ are excluded since we wouldn't have a package for those values. Also, the height and radius of the package must be greater than zero, so we have the graph below.



The volume of a right circular cylinder is $V = \pi r^2 h$. In addition, $h \leq {}^{-}2\pi r + 108$. To simplify the calculation, substitute $h = {}^{-}2\pi r + 108$ to find the volume when the height and the girth is equal to 108 inches.

$V(r) = \pi r^2({}^{-}2\pi r + 108) = {}^{-}2\pi^2 r^3 + 108\pi r^2$

This is a cubic polynomial function in r. The graph of the volume of the packages that the shipping company will allow is represented by the region bound by this curve and the x- and y-axes.

(continued on next page)

Sample Performance Assignment Response: Score Point 4 *(continued)*

The domain of V is $0 < r < \frac{54}{\pi}$.

$V\left(\frac{54}{\pi}\right) = {}^{-}2\pi^2\left(\frac{54}{\pi}\right)^3 + 108\pi\left(\frac{54}{\pi}\right)^2 = 0$ cubic inches

V has zeros at $r = 0$ and $r = \frac{54}{\pi}$. These numbers are not in the domain of V and will be indicated on the graph by open circles.

Use the first and second derivative tests to graph this function.

$V' = {}^{-}6\pi^2 r^2 + 216\pi r$

$0 = {}^{-}6\pi^2 r^2 + 216\pi r$

$r = 0$ is one solution, and

$6\pi r = 216$

$r = \frac{36}{\pi}$

$V\left(\frac{36}{\pi}\right) = {}^{-}2\pi^2\left(\frac{36}{\pi}\right)^3 + 108\pi\left(\frac{36}{\pi}\right)^2 = \left(\frac{36}{\pi}\right) = 14851.1$ cubic inches

In the domain of interest, V has a critical point at $r = \frac{36}{\pi}$ and $V = 14851.1$ cubic inches.

$V'' = {}^{-}12\pi^2 r + 216\pi$

$0 = {}^{-}12\pi^2 r + 216\pi$

$r = \frac{18}{\pi} = 5.73$

$V\left(\frac{18}{\pi}\right) = {}^{-}2\pi^2\left(\frac{18}{\pi}\right)^3 + 108\pi\left(\frac{18}{\pi}\right)^2 = \frac{72\,(18)^2}{\pi} = 7425.5$ cubic inches

V has a point of inflexion at $r = 18/\pi =$ inches and $V = 7425.5$ cubic inches.

It is concave up on $0 \le r < \frac{18}{\pi}$ and concave down on $\frac{18}{\pi} < r \le \frac{54}{\pi}$.

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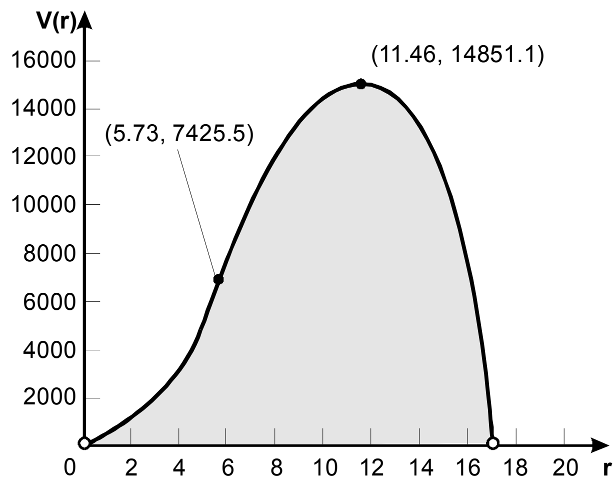
Sample Performance Assignment Response: Score Point 4 *(continued)*

The graph below summarizes these results.



The shaded region represents the volume and radii of the packages that the shipping company will accept for delivery. The shaded region is represented by the inequality $V \leq {}^{-}2\pi^2 r^3 + 108\pi r^2$.

The dimension of the cylinder of maximum volume can be determined by substituting $r = \frac{36}{\pi}$ into $h = {}^{-}2\pi r + 108$, or $h = {}^{-}2\pi\left(\frac{36}{\pi}\right) + 108 = 36$ inches. Hence the dimensions of the cylindrical package of maximum volume are a radius of about 11.46 inches and a height of 36 inches. Note that if we calculate the volume of this cylinder from $V = \pi r^2 h$, we get $V = \pi\left(\frac{36}{\pi}\right)^2(36) = 14851.1$ cubic inches, which is in agreement with the result found above.

ANSWER KEY

Field 10: Mathematics

Question Number	Correct Response	Objective
1.	**B**	Understand principles and concepts related to integers, fractions, decimals, percents, ratios, and proportions and their application to problem solving.
2.	**A**	Understand principles and concepts related to integers, fractions, decimals, percents, ratios, and proportions and their application to problem solving.
3.	**B**	Understand principles and concepts related to integers, fractions, decimals, percents, ratios, and proportions and their application to problem solving.
4.	**D**	Understand the properties of the real and complex number systems, and solve problems related to their structure.
5.	**B**	Understand the properties of the real and complex number systems, and solve problems related to their structure.
6.	**D**	Understand the properties of the real and complex number systems, and solve problems related to their structure.
7.	**C**	Understand the principles of number theory.
8.	**D**	Understand the principles of number theory.
9.	**A**	Understand the principles and properties of discrete mathematics and the application of discrete mathematics to problem solving.
10.	**B**	Understand the principles and properties of discrete mathematics and the application of discrete mathematics to problem solving.
11.	**B**	Understand the principles and properties of discrete mathematics and the application of discrete mathematics to problem solving.
12.	**D**	Understand principles and concepts of descriptive statistics and their application to the problem-solving process.
13.	**B**	Understand principles and concepts of descriptive statistics and their application to the problem-solving process.
14.	**C**	Understand principles and concepts of descriptive statistics and their application to the problem-solving process.
15.	**A**	Understand principles and concepts of descriptive statistics and their application to the problem-solving process.
16.	**D**	Understand the fundamental principles of probability and probability distributions.
17.	**C**	Understand the fundamental principles of probability and probability distributions.
18.	**B**	Use techniques related to probability and probability distributions to analyze real-world situations.
19.	**A**	Use techniques related to probability and probability distributions to analyze real-world situations.
20.	**B**	Use techniques related to probability and probability distributions to analyze real-world situations.

(continued on next page)

Question Number	Correct Response	Objective
21.	**A**	Understand methods used in collecting, reporting, and analyzing data.
22.	**D**	Understand methods used in collecting, reporting, and analyzing data.
23.	**A**	Understand algebraic functions, relations, and expressions.
24.	**B**	Understand algebraic functions, relations, and expressions.
25.	**D**	Understand algebraic functions, relations, and expressions.
26.	**A**	Understand the principles and properties of linear and matrix algebra.
27.	**C**	Understand the principles and properties of linear and matrix algebra.
28.	**B**	Understand the principles and properties of linear and matrix algebra.
29.	**D**	Understand the properties of linear and quadratic equations, inequalities, and functions and their application to the problem-solving process.
30.	**C**	Understand the properties of linear and quadratic equations, inequalities, and functions and their application to the problem-solving process.
31.	**A**	Understand the properties of linear and quadratic equations, inequalities, and functions and their application to the problem-solving process.
32.	**A**	Understand radical, exponential, and logarithmic functions and their application to the problem-solving process.
33.	**C**	Understand radical, exponential, and logarithmic functions and their application to the problem-solving process.
34.	**B**	Understand radical, exponential, and logarithmic functions and their application to the problem-solving process.
35.	**C**	Understand and interpret polynomial, rational, and absolute value functions and relations.
36.	**A**	Understand and interpret polynomial, rational, and absolute value functions and relations.
37.	**A**	Understand and interpret polynomial, rational, and absolute value functions and relations.
38.	**A**	Understand principles, concepts, and procedures related to measurement.
39.	**C**	Understand principles, concepts, and procedures related to measurement.
40.	**B**	Understand principles, concepts, and procedures related to measurement.
41.	**C**	Understand properties of geometric figures and their application to the problem-solving process.
42.	**A**	Understand properties of geometric figures and their application to the problem-solving process.
43.	**B**	Understand properties of geometric figures and their application to the problem-solving process.

(continued on next page)

Question Number	Correct Response	Objective
44.	**B**	Understand and interpret drawings of three-dimensional objects.
45.	**D**	Understand and interpret drawings of three-dimensional objects.
46.	**C**	Understand and interpret drawings of three-dimensional objects.
47.	**C**	Understand and interpret drawings of three-dimensional objects.
48.	**A**	Understand the principles and properties of coordinate geometry and the connections between geometry and algebra.
49.	**B**	Understand the principles and properties of coordinate geometry and the connections between geometry and algebra.
50.	**A**	Understand the principles of vectors and transformational geometry and the application of transformational geometry to the problem-solving process.
51.	**D**	Understand the principles of vectors and transformational geometry and the application of transformational geometry to the problem-solving process.
52.	**B**	Understand the principles of vectors and transformational geometry and the application of transformational geometry to the problem-solving process.
53.	**C**	Understand techniques used to model and solve problems related to triangles.
54.	**D**	Understand techniques used to model and solve problems related to triangles.
55.	**B**	Understand techniques used to model and solve problems related to triangles.
56.	**D**	Understand the properties of trigonometric functions and identities.
57.	**B**	Understand the properties of trigonometric functions and identities.
58.	**C**	Understand the properties of trigonometric functions and identities.
59.	**C**	Understand characteristics and applications of the concepts of limit, continuity, and rate of change.
60.	**B**	Understand characteristics and applications of the concepts of limit, continuity, and rate of change.
61.	**D**	Understand the derivative of a function and its applications to the problem-solving process.
62.	**A**	Understand the derivative of a function and its applications to the problem-solving process.
63.	**B**	Understand the derivative of a function and its applications to the problem-solving process.
64.	**C**	Understand the integral of a function and its applications to the problem-solving process.
65.	**A**	Understand the integral of a function and its applications to the problem-solving process.
66.	**D**	Understand the integral of a function and its applications to the problem-solving process.

(continued on next page)

Question Number	Correct Response	Objective
67.	**D**	Understand principles of problem solving, and apply varied and efficient problem-solving techniques and strategies and technological tools to explore and solve problems in context.
68.	**D**	Understand principles of problem solving, and apply varied and efficient problem-solving techniques and strategies and technological tools to explore and solve problems in context.
69.	**D**	Understand principles of problem solving, and apply varied and efficient problem-solving techniques and strategies and technological tools to explore and solve problems in context.
70.	**C**	Understand mathematical communication and the use of mathematical terminology, symbols, and representations to communicate information.
71.	**B**	Understand mathematical communication and the use of mathematical terminology, symbols, and representations to communicate information.
72.	**D**	Understand mathematical communication and the use of mathematical terminology, symbols, and representations to communicate information.
73.	**B**	Understand mathematical reasoning, and apply techniques of mathematical reasoning in varied contexts.
74.	**C**	Understand mathematical reasoning, and apply techniques of mathematical reasoning in varied contexts.
75.	**D**	Understand mathematical reasoning, and apply techniques of mathematical reasoning in varied contexts.
76.	**B**	Understand the connections among the domains of mathematics and the relationships between mathematics and other disciplines and real-world situations.
77.	**D**	Understand the connections among the domains of mathematics and the relationships between mathematics and other disciplines and real-world situations.
78.	**B**	Understand the connections among the domains of mathematics and the relationships between mathematics and other disciplines and real-world situations.
79.	**D**	Understand the nature and structure of axiomatic systems.
80.	**A**	Understand the nature and structure of axiomatic systems.

PREPARATION RESOURCES

Field 10: Mathematics

The resources listed below may help you prepare for the AEPA test in this field. These preparation resources have been identified by content experts in the field to provide up-to-date information that relates to the field in general. You may wish to use current issues or editions to obtain information on specific topics for study and review.

Journals

American Mathematical Monthly, Mathematical Association of America.

Mathematics Magazine, Mathematical Association of America.

Mathematics Teacher, National Council of Teachers of Mathematics.

Other Resources

Lay, D.C. (2003). *Linear Algebra and Its Applications* (3rd ed.). Boston, MA: Addison-Wesley.

Demana, F., Waits, B. K., Clemens, S. R., Foley, G. D, and Kennedy, D. (2003). *Precalculus: Functions and Graphs* (5th ed.). Menlo Park, CA: Addison-Wesley.

Dugopolski, M. (2002). *College Algebra* (3rd ed.). Boston, MA: Addison-Wesley.

Faires, J. D. and DeFranza, J. (2003). *Precalculus* (3rd ed.). Stamford, CT: Thomson–Brooks/Cole.

Larson, Boswell, Stiff. (2001). *Geometry* (10th ed.). Geneva, IL: Mcdougal Littell–Houghton Mifflin.

Hungerford, T. W. (2001). *Contemporary College Algebra and Trigonometry: A Graphing Approach*. Philadelphia, PA: Harcourt College Publishers.

Moore, D. S. and McCabe, G. P. (2003). *Introduction to the Practice of Statistics* (4th ed.). New York, NY: W.H. Freeman.

National Council of Teachers of Mathematics. (2000). *Principles and Standards for School Mathematics.* Reston, VA: The National Council of Teachers of Mathematics, Inc.

National Council of Teachers of Mathematics. (1991). *Professional Standards for Teaching Mathematics.* Reston, VA: The National Council of Teachers of Mathematics, Inc.

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Stewart, J. (2000). *Calculus—Concepts and Contexts* (2nd ed.). Stamford, CT: Thomson-Brooks/Cole.

Triola, M. F. (2003). *Elementary Statistics* (9th ed.). Boston, MA: Addison Wesley Longman, Inc.

Online Sources

Arizona Department of Education, Mathematics Standards,
http://www.ade.state.az.us/standards/math/articulated.asp